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Parametric Macromodeling Based on Amplitude and Frequency Scaled Systems with Guaranteed Passivity

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SUMMARY

We propose a novel parametric macromodeling method for systems described by scattering parameters, which depend on multiple design variables such as geometrical layout or substrate features. It is able to build accurate multivariate macromodels that are stable and passive over the entire design space. Poles and residues are parameterized indirectly. The proposed method is based on an efficient and reliable combination of rational identification, a procedure to find amplitude and frequency scaling

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system coefficients, and positive interpolation schemes. Pertinent numerical examples validate the proposed parametric macromodeling technique.

KEY WORDS: Parametric macromodeling, rational approximation, interpolation, passivity.

1. Introduction

Design space exploration, design optimization and sensitivity analysis of electromagnetic (EM) systems are usually performed during a typical design process that consequently requires multiple frequency-domain simulations for different design parameter values (e.g. layout features). Parametric macromodels are suitable to efficiently and accurately perform these design activities, while using multiple EM simulations may often be not feasible due to the high computational cost per simulation. Parametric macromodels approximate the complex behavior of EM systems, which is typically characterized by the frequency (or time) and several design parameters, such as geometrical or substrate features.

Different parametric macromodeling techniques have been developed over the last years. Both poles and residues are parameterized in some approaches [1, 2] and this full parameterization allows to model dynamic multivariate data samples. Unfortunately, these techniques are not able to guarantee overall stability and passivity of parametric macromodels. In other formulations, only residues are parameterized [3–5]; this reduces the modeling capability compared with techniques [1, 2], but guarantees the stability and passivity of parametric macromodels. Recently, new parametric macromodeling methods in [6–8] parameterize poles and residues and are able to guarantee stability and passivity over the entire design space, thereby overcoming some limitations of [1, 2] and [3–5]. Unfortunately, these methods can only deal with rational models of the same order and are sensitive to the issues related to the interpolation of state-space matrices [9]. A passivity preserving interpolation of state-space matrices is performed by means of the matrix solution of positive-real and bounded-real lemma, which can be solved using Linear Matrix Inequalities (LMI) or Riccati equation solvers. While the LMI solvers of [10] are significantly faster than classical convex optimization algorithms, the complexity of LMI computations can grow quickly with the number of states n. For example, the number of operations required to solve a Riccati equation is $O(n^3)$, while the cost of solving an equivalent LMI is $O(n^6)$ [11]. The method proposed in [8] uses a final optimization procedure with the aim of tuning the accuracy of the parametric macromodel. This optimization step can be computationally expensive and is only described for one parameter in addition to frequency.

This paper proposes a new parametric macromodeling technique for scattering ($\mathbf{S}$) representations, which indirectly parameterizes poles and residues and is able to guarantee stability and passivity over the entire design space. No solution of positive-real and bounded-real lemma is required to ensure overall stability and passivity. The Vector Fitting (VF) technique [12] is initially used to build a set of univariate frequency-dependent macromodels for different combinations of the design variables, as in [3–5]. These initial univariate macromodels are called *root macromodels*. Stability for each *root macromodel* is enforced by pole-flipping [12], while passivity is checked and enforced by means of standard techniques (see e.g. [13,14]). Next, amplitude and frequency scaling system coefficients are computed for each *root macromodel* and properly parameterized by positive interpolation operators [15], [16]. Finally, a parametric macromodel is obtained by a combination of *root macromodels* and corresponding amplitude and frequency scaling coefficients, using positive interpolation schemes that preserve stability and passivity over the complete design space. The proposed technique can deal with *root macromodels* of different orders, it is able to guarantee overall passivity and stability without solving positive-real and bounded-real lemma and is not sensitive to the issues related to the interpolation of state-space matrices [9]. Some pertinent numerical examples validate the proposed method.

2. Parametric Macromodeling

The goal of the present algorithm is to build a parametric representation $\mathbf{R}(s, \boldsymbol{g})$ that accurately models a set of data samples $\{(s, \boldsymbol{g})_k, \mathbf{H}(s, \boldsymbol{g})_k\}_{k=1}^{K_{tot}}$ and guarantees stability and passivity over the entire design space. These data samples depend on the complex frequency $s = j\omega$ and several design variables $\boldsymbol{g} = (g^{(m)})_{m=1}^{M}$, such as layout features or substrate parameters. A parametric macromodel in the form

$$\mathbf{R}(s, \boldsymbol{g}) = \mathbf{C}(\boldsymbol{g}) \left(s\mathbf{I} - \mathbf{A}(\boldsymbol{g})\right)^{-1} \mathbf{B}(\boldsymbol{g}) + \mathbf{D}(\boldsymbol{g}) \tag{1}$$

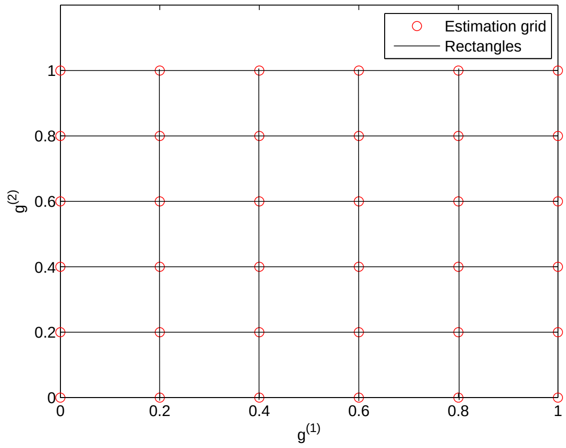
is computed by the proposed parametric macromodeling method. The design space $\mathcal{D}(\boldsymbol{g})$ is considered as the parameter space $\mathcal{P}(s, \boldsymbol{g})$ without frequency. The parameter space $\mathcal{P}(s, \boldsymbol{g})$ contains all parameters $(s, \boldsymbol{g})$. If the parameter space is (M+1)-dimensional, the design space is M-dimensional. Two data grids are used in the modeling process: an estimation grid and a validation grid. The estimation grid is utilized to build the *root macromodels*. The validation grid is used to validate the modeling capability of the parametric macromodel in a set of points of the design space previously not used for the construction of the *root macromodels*.

2.1. Root Macromodels

Starting from a set of data samples $\{(s, \boldsymbol{g})_k, \mathbf{H}(s, \boldsymbol{g})_k\}_{k=1}^{K_{tot}}$, a set of frequency-dependent rational macromodels is built for some design space points by means of the VF technique [12]. A pole-flipping scheme is used to enforce stability [12], while passivity assessment and enforcement can be accomplished using the robust standard techniques [13, 14]. The result of this initial procedure is a set of rational univariate macromodels, stable and passive, that we call *root macromodels*. We note that each *root macromodel* can have a different order. These are the starting points to build a parametric macromodel.

2.2. Amplitude and Frequency Scaling Coefficients

Once the *root macromodels* are available, the next step is gluing them together and building a multivariate representation $\mathbf{R}(s, \boldsymbol{g})$ which models the set of data samples $\{(s, \boldsymbol{g})_k, \mathbf{H}(s, \boldsymbol{g})_k\}_{k=1}^{K_{tot}}$ and preserves stability and passivity over the entire design space. The design space is divided into cells using hyperrectangles (regular grid) [17] or simplices (regular and scattered grid) [18, 19]. The process of dividing the design space into simplices is called triangulation in 2-D and tessellation in higher dimensions. A simplex, or N-simplex, is the N-D analogue of a triangle in 2-D and a tetrahedron in 3-D, while a hyperrectangle is the N-D analogue of a rectangle in 2-D and a rectangular parallelepiped in 3-D. For each data distribution many tessellations can be constructed. The minimal requirement is that the simplices do not overlap, and that there are no holes. Kuhn tessellation [18] can be used for a regular design space grid, this technique splits every hypercube in $\mathbb{R}^{\mathrm{N}}$ into N! simplices. Delaunay tessellation [19] can be used for an irregular design space grid, this technique is a space-filling aggregate of simplices and can be performed using standard algorithms [20]. Fig. 1 shows a possible 2-D design space divided into cells, in the regular and scattered case, respectively.



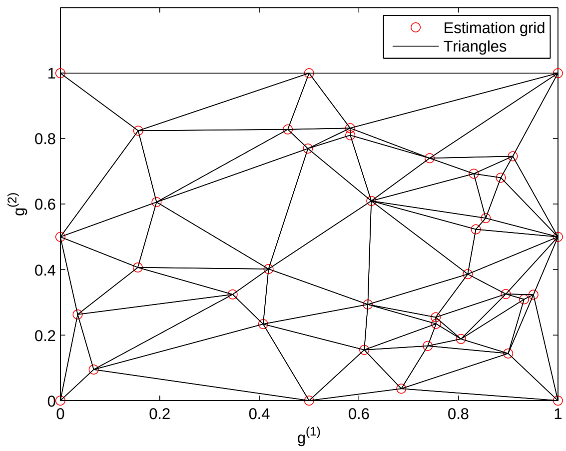


Figure 1. Design space divided into cells: regular (top) and scattered (bottom).

Once the design space is divided into cells, a local parametric macromodel is associated to every cell that is a subdomain of the entire design space. We indicate a cell region of the design space as $\Omega_i,\ i = 1, ..., P$ and the corresponding vertices as $\boldsymbol{g}_k^{\Omega_i},\ k = 1, ..., Q$. We note that each vertex corresponds to a *root macromodel* $\mathbf{R}(s, \boldsymbol{g}_k^{\Omega_i})$. For each cell (a subdomain) of the design space, an optimization procedure is used to find the amplitude and frequency scaling system coefficients that make each vertex an accurate approximant of the other cell vertices. For each vertex $\mathbf{R}(s, \boldsymbol{g}_k^{\Omega_i})$, a set of amplitude $\alpha_k(\boldsymbol{g}_j^{\Omega_i}),\ j = 1, \ldots, Q$ and frequency scaling $\beta_k(\boldsymbol{g}_j^{\Omega_i}),\ j = 1, \ldots, Q$ real coefficients are found by means of optimization, such that

$$\alpha_k(\boldsymbol{g}_j^{\Omega_i})\mathbf{R}(s\beta_k(\boldsymbol{g}_j^{\Omega_i}), \boldsymbol{g}_k^{\Omega_i}) \simeq \mathbf{R}(s, \boldsymbol{g}_j^{\Omega_i}),\ j \neq k \tag{2}$$

$$\alpha_k(\boldsymbol{g}_j^{\Omega_i}) = \beta_k(\boldsymbol{g}_j^{\Omega_i}) = 1,\ j = k \tag{3}$$

Then, if the response of the system under modeling needs to be computed in a specific design space point $\widehat{\boldsymbol{g}}$, a subdomain that contains $\widehat{\boldsymbol{g}}$ is to be found. For each vertex *root macromodel* of the found subdomain, the corresponding sets of amplitude and frequency scaling coefficients $\alpha_k(\boldsymbol{g}_j^{\Omega_i}), \beta_k(\boldsymbol{g}_j^{\Omega_i})$ are interpolated in $\widehat{\boldsymbol{g}}$ and a rational model $\widehat{\alpha}_k\mathbf{R}(s\widehat{\beta}_k, \boldsymbol{g}_k^{\Omega_i})$ is built, where $\widehat{\alpha}_k = \alpha_k(\widehat{\boldsymbol{g}})$ and $\widehat{\beta}_k = \beta_k(\widehat{\boldsymbol{g}})$. Finally, the set of modified *root macromodels* $\widehat{\alpha}_k\mathbf{R}(s\widehat{\beta}_k, \boldsymbol{g}_k^{\Omega_i}),\ k = 1, ..., Q$, is interpolated at an input/output level as described in [3–5]. We note that if a generic *root macromodel* $\mathbf{R}(s, \boldsymbol{g}_k^{\Omega_i})$ has the state-space representation $\{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}\}$, then a corresponding amplitude and frequency scaled version $\widehat{\alpha}_k\mathbf{R}(s\widehat{\beta}_k, \boldsymbol{g}_k^{\Omega_i})$ has the state-space representation $\{\widetilde{\mathbf{A}}, \widetilde{\mathbf{B}}, \widetilde{\mathbf{C}}, \widetilde{\mathbf{D}}\}$ with

$$\begin{aligned}
\widetilde{\mathbf{A}} &= (\widehat{\beta}_k)^{-1}\mathbf{A} \\
\widetilde{\mathbf{B}} &= \mathbf{B} \\
\widetilde{\mathbf{C}} &= \widehat{\alpha}_k(\widehat{\beta}_k)^{-1}\mathbf{C} \\
\widetilde{\mathbf{D}} &= \widehat{\alpha}_k\mathbf{D}
\end{aligned} \tag{4}$$

2.3. Multivariate Interpolation of Amplitude, Frequency Scaling Coefficients and Root Macromodels

It is known that, while a passive system is also stable, the reverse is not necessarily true [21]. Passivity is crucial when the macromodel is utilized in a circuit simulator (e.g. SPICE [22]) for transient analysis. Passivity refers to the property of systems that cannot generate more energy than they absorb through their electrical ports. When the system is terminated on any arbitrary passive loads, none of them will cause the system to become unstable [23]. The passivity of scattering input-output representations is also called nonexpansivity [24]. A linear network described by scattering matrix $\mathbf{S}(s)$ is passive if [25]:

1. $\mathbf{S}(s^*) = \mathbf{S}^*(s)$ for all s, where "$*$" is the complex conjugate operator.

2. $\mathbf{S}(s)$ is analytic in $\Re e(s) > 0$.

3. $\mathbf{I} - \mathbf{S}^{\mathrm{T}}(s^*)\mathbf{S}(s) \geq 0$; $\forall s : \Re e(s) > 0$.

Condition 3) for nonexpansivity is equivalent to the condition $\|\mathbf{S}(s)\|_\infty \leq 1$ ($\mathbf{H}_\infty$ norm) [24], i.e., the largest singular value of $\mathbf{S}(s)$ does not exceed one in the right-half s-plane. The interpolated amplitude and frequency scaling real coefficients $\alpha_k(\boldsymbol{g}), \beta_k(\boldsymbol{g})$ have to satisfy the conditions

$$0 \leq \alpha_k(\boldsymbol{g}) \leq 1 \tag{5a}$$

$$\beta_k(\boldsymbol{g}) \geq 0 \tag{5b}$$

to guarantee the passivity of each *root macromodel* $\alpha_k(\boldsymbol{g})\mathbf{R}(s\beta_k(\boldsymbol{g}), \boldsymbol{g}_k^{\Omega_i})$.

Multivariate interpolation based on a class of positive interpolation operators [15], [16] is used to parameterize $\alpha_k(\boldsymbol{g}), \beta_k(\boldsymbol{g})$. It is able to preserve the passivity of each amplitude and frequency scaled *root macromodel* over the entire design space by guaranteeing the properties (5a)-(5b). The same positive multivariate interpolation schemes are used to interpolate the set of modified *root macromodels* $\widehat{\alpha}_k \mathbf{R}(s\widehat{\beta}_k, \boldsymbol{g}_k^{\Omega_i})$, $k = 1, ..., Q$, at an input/output level, which results in a parametric macromodel, stable and passive over the entire design space.

Multivariate interpolation can be realized by means of tensor product [26] or tessellation [19] methods. Tensor product multivariate interpolation methods require the data points distributed on a fully filled, but not necessarily equidistant, rectangular grid, while tessellation-based multivariate interpolation methods can handle scattered or irregularly distributed data points. Any interpolation scheme based on a class of positive interpolation operators can be used.

In the bivariate case (s, g), each interpolated function $\mathbf{T}(g)$, that can be amplitude, frequency scaling coefficients and *root macromodels*, can be written as

$$\mathbf{T}(g) = \sum_{k=1}^{K_1} \mathbf{T}_{g_k} \ell_k(g) \tag{6}$$

where K_1 represents the number of *root macromodels* and each interpolation kernel $\ell_k(g)$ is a scalar function satisfying the following constraints

$$0 \leq \ell_k(g) \leq 1, \tag{7}$$

$$\ell_k(g_i) = \delta_{k,i}, \tag{8}$$

$$\textstyle\sum_{k=1}^{K_1} \ell_k(g) = 1. \tag{9}$$

A possible choice is to select $\ell_k(g)$ as in piecewise linear interpolation

$$\frac{g-g_{k-1}}{g_k-g_{k-1}},\ g \in [g_{k-1}, g_k]\,,\ k = 2, ..., K_1, \tag{10}$$

$$\frac{g_{k+1}-g}{g_{k+1}-g_k},\ g \in [g_k, g_{k+1}]\,,\ k = 1, ..., K_1 - 1, \tag{11}$$

$$0 \qquad\qquad , \text{otherwise} \tag{12}$$

The bivariate formulation can easily be generalized to the multivariate case by using multivariate interpolation methods that belong to the general class of positive interpolation operators: e.g., the piecewise multilinear and multivariate simplicial methods [17]. We note that the interpolation kernel functions of these methods only depend on the design space grid points and their computation does not require the solution of a linear system to impose an interpolation constraint. In the case of piecewise multilinear interpolation, each interpolated function $\mathbf{T}(g^{(1)}, ..., g^{(M)})$ can be written as

$$\begin{aligned} &\mathbf{T}(g^{(1)}, ..., g^{(M)}) = \\ &= \sum_{k_1=1}^{K_1} \cdots \sum_{k_M=1}^{K_M} \mathbf{T}_{\left(g^{(1)}_{k_1}, ..., g^{(M)}_{k_M}\right)} \ell_{k_1}(g^{(1)}) \cdots \ell_{k_M}(g^{(M)}) \end{aligned} \tag{13}$$

where each $\ell_{k_i}(g^{(i)}),\ i = 1, ..., M$ satisfies constraints (7)-(9) and is selected as in piecewise linear interpolation. Concerning multivariate scattered interpolation, a tessellation-based linear interpolation scheme can be used [5] to build a parametric macromodel. This interpolation method belongs to the class of positive interpolation operators. These positive interpolation schemes have been already used in [3–5], where a parametric macromodel is built by interpolating a set of *root macromodels* treated as input-output systems, while preserving overall stability and passivity. In the proposed new parametric macromodeling technique, a powerful novelty is the use of some interpolated amplitude and frequency scaling system coefficients. It allows to parameterize poles and residues indirectly, hence the modeling capability of the proposed algorithm is increased with respect to [3–5], where the interpolation process were only applied to the *root macromodels*, and therefore only residues were parameterized.

2.4. Passivity Preserving Interpolation of Amplitude and Frequency Scaling Coefficients

In this section, we prove that a passive system $\mathbf{R}(s)$ remains passive if an amplitude scaling coefficient α and a frequency scaling coefficient β, which satisfy the properties (5a)-(5b), are applied to it. The amplitude scaling coefficient α is a multiplicative factor at the input/output level of the system, while the frequency scaling coefficient β is a compression or expansion term for the Laplace variable s. It is easy to prove that if β satisfies (5b), passivity is preserved, and that if α satisfies (5a), the first two conditions for passivity are preserved. Concerning α and the third passivity condition

$$\|\alpha\mathbf{R}(\beta s)\|_\infty = \alpha\|\mathbf{R}(\beta s)\|_\infty \leq \alpha \leq 1 \tag{14}$$

Therefore, if α satisfies (5a), passivity is preserved.

2.5. Passivity Preserving Interpolation of Root Macromodels

Once a set of amplitude and frequency scaled *root macromodels* $\widehat{\alpha}_k\mathbf{R}(s\widehat{\beta}_k, \boldsymbol{g}_k^{\Omega_i}),\ k = 1, \ldots, Q$, which are stable and passive, is built for each cell of the design space, the next step of the proposed method is focused on gluing together these *root macromodels* by a multivariate interpolation scheme to obtain a parametric macromodel $\mathbf{R}(s, \boldsymbol{g})$ with overall stability and passivity. Condition 1) is preserved in (6) and the proposed multivariate extensions, as they are weighted sums with real nonnegative weights of systems respecting this first condition. Condition 2) is preserved in (6) and the proposed multivariate extensions, as they are weighted sums of strictly stable rational macromodels. Condition 3) is equivalent to the condition $\|\mathbf{R}(s)\|_\infty \leq 1$ ($\mathbf{H}_\infty$ norm) [24], therefore in the bivariate case we can write:

$$\|\mathbf{R}(s,g)\|_\infty \leq \sum_{k=1}^{K_1} \|\mathbf{R}(s,g_k)\|_\infty \, \ell_k(g) \leq \sum_{k=1}^{K_1} \ell_k(g) = 1 \tag{15}$$

Similar results are obtained for the proposed multivariate cases, so condition 3) is satisfied by construction. We have demonstrated that all three passivity conditions for scattering representations are preserved in the proposed parametric macromodeling algorithm.

3. Numerical examples

This section presents two numerical examples to validate the proposed parametric macromodeling approach on application cases. Let us define the absolute error

$$Err(\boldsymbol{g}) = max\Big(\Big|R_{i,j}(s_k,\boldsymbol{g}) - H_{i,j}(s_k,\boldsymbol{g})\Big|\Big) \tag{16}$$
$$i = 1,\ldots,P_{in},\;\; j = 1,\ldots,P_{out},\;\; k = 1,\ldots,K_s$$

where P_{in} and P_{out} are the number of inputs and outputs of the system, respectively, and K_s is equal to the number of frequency samples. The worst case absolute error over the validation grid is chosen to assess the accuracy and the quality of parametric macromodels

$$\boldsymbol{g}_{max} = \underset{\boldsymbol{g}}{\text{argmax}}\, Err(\boldsymbol{g}),\ \ \boldsymbol{g} \in validation\ grid \tag{17}$$

$$Err_{max} = Err(\boldsymbol{g}_{max}) \tag{18}$$

The number of poles for each *root macromodel* is selected adaptively in VF by a bottom-up approach, in such a way that the corresponding maximum absolute error is smaller than -60 dB. The proposed macromodeling algorithm is compared with the technique described in [4,5], to confirm its enhanced modeling capability and accuracy.

3.1. Spiral inductor with variable outer area

An integrated spiral inductor has been modeled in this example. The structure is shown in Fig. 2. The conductors width is equal to 46 μm. A bivariate macromodel is built as a function of the outer area A of the spiral inductor in addition to frequency. Their corresponding ranges are shown in Table I.

Table I. Parameters of the spiral inductor.

Parameter	Min	Max
Frequency ($freq$)	10 kHz	33 GHz
Outer area (A)	460 μm^2	930 μm^2

The scattering parameters have been computed by means of an EM solver based on the Partial Element Equivalent Circuit method [27,28] over a validation grid of 250×15 samples, for frequency and outer area respectively. We have built *root macromodels* for 8 values of the

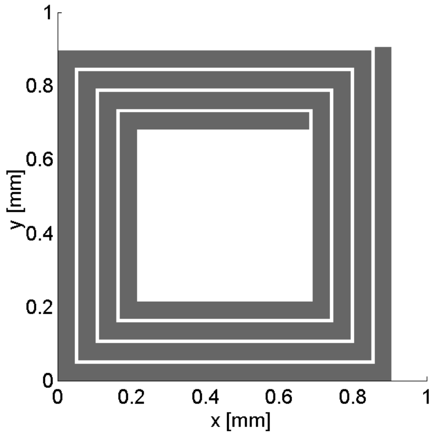


Figure 2. Structure of the spiral inductor.

outer area by means of VF, each with an order chosen by the error-based bottom-up approach described in Section 3. Since the data under modeling are highly dynamic, *root macromodels* with different orders are required, as shown in Table II. The passivity of each model has been verified by checking the eigenvalues of the corresponding Hamiltonian matrix [24] and enforced, if needed. Finally, a bivariate macromodel is obtained as explained in Section 2, using linear interpolation for the amplitude and frequency scaling coefficients, and *root macromodels*. Fig. 3 shows the magnitude of the parametric macromodel of $\mathbf{S}_{11}(s, A)$. Figs. 4-5 compare $\mathbf{S}_{11}(s, A)$ and its macromodel for the outer area values $A = \{495, 896\}$ $\mu\mathrm{m}^2$ that have not been used for the generation of the *root macromodels*. The worst case absolute error defined in (18) is equal to 2.71 dB and -50 dB for the old [4,5] and new parametric macromodeling techniques, respectively.

As clearly seen, a very good agreement is obtained between the original data and the new proposed parametric macromodeling technique that achieves a better accuracy and modeling

Table II. Number of poles of *root macromodels*.

A_1	8
A_2	10
A_3	10
A_4	10
A_5	10
A_6	12
A_7	14
A_8	14

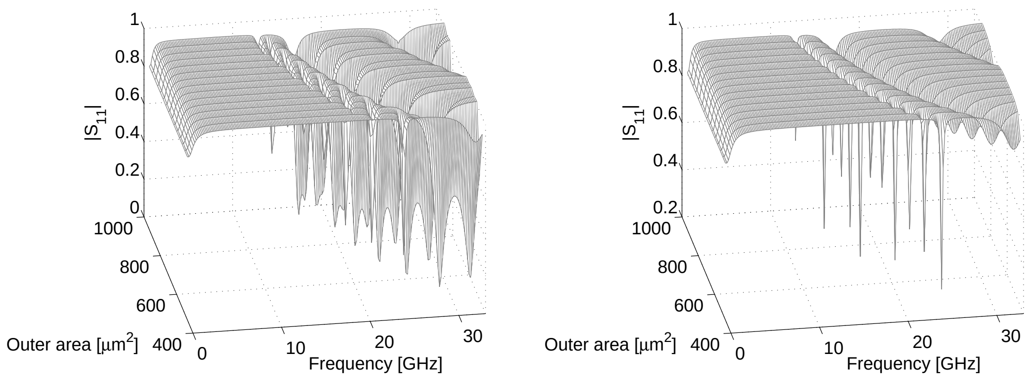


Figure 3. Magnitude of the bivariate macromodel of $\mathbf{S}_{11}(s, A)$ by means of the old technique [4, 5] (left) and the new technique (right).

capability with respect to [4, 5]. The new parametric macromodeling method captures the behavior of the system very accurately, while preserving stability and passivity over the entire design space.

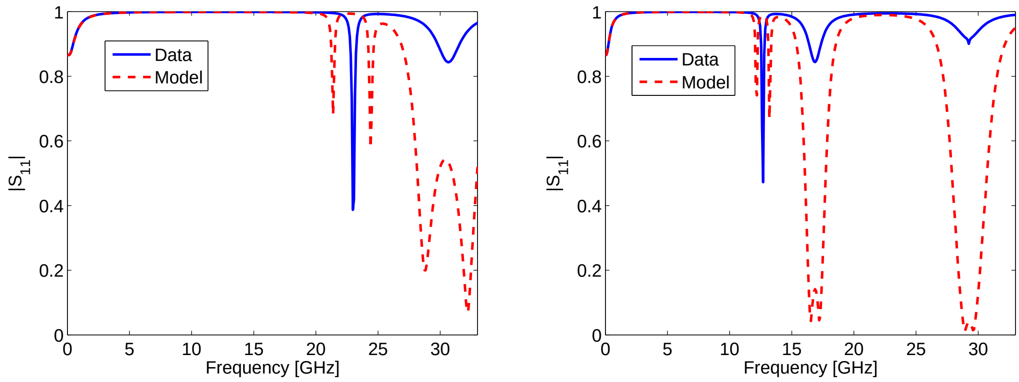


Figure 4. Magnitude of the bivariate macromodel of $\mathbf{S}_{11}(s, A)$ ($A = \{476, 915\}$ μm^2) (old technique [4, 5]).

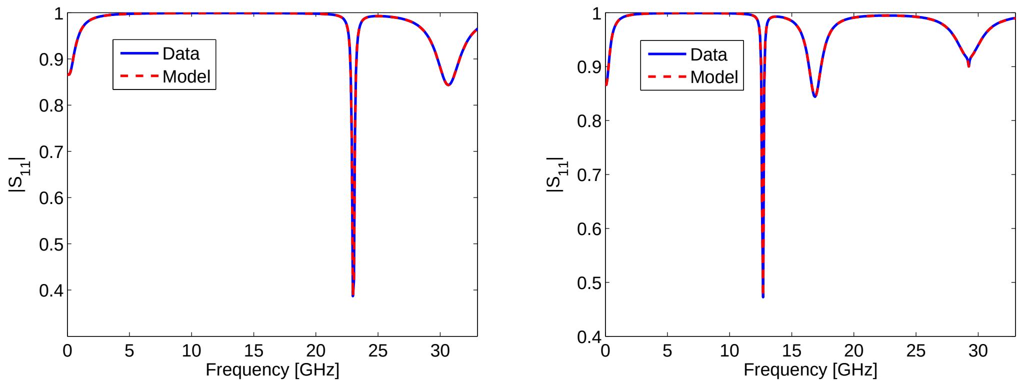


Figure 5. Magnitude of the bivariate macromodel of $\mathbf{S}_{11}(s, A)$ ($A = \{476, 915\}$ μm^2) (new proposed technique).

3.2. Three coupled microstrips with variable length and spacing

In this second example, three coupled microstrips with frequency-dependent per-unit-length parameters have been modeled. The cross section is shown in Fig. 6.

The conductors have width $w = 100$ μm and thickness $t = 50$ μm. The dielectric is 300

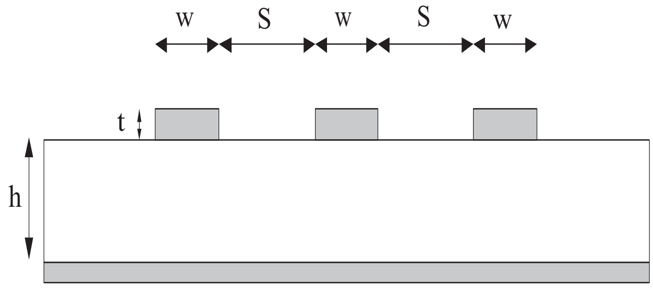


Figure 6. Cross section of the three coupled microstrips.

μm thick and characterized by a dispersive and lossy permittivity which has been modeled by the wideband Debye model [29]. The frequency-dependent per-unit-length parameters have been evaluated using a commercial tool [30]. The length L of the microstrips and the spacing S between the microstrips are considered as parameters in addition to frequency. Their corresponding ranges are shown in Table III.

Table III. Design parameters of the three coupled microstrips structure.

Parameter	Min	Max
Frequency (freq)	100 kHz	15 GHz
Length (L)	10 mm	20 mm
Spacing (S)	200 μm	400 μm

The scattering parameters have been computed by means of the exact transmission line theory (TLT) [31] starting from the per-unit-length parameters over a validation grid of $250 \times 13 \times 11$ samples, for frequency, length and spacing, respectively. We have built *root macromodels* for 7 values of the length and 6 values of the spacing by means of VF, each with an order chosen by the error-based bottom-up approach described in Section 3. Similarly to the previous example, *root macromodels* with different orders are required, as shown in Table IV. Finally, a trivariate macromodel is obtained as explained in Section 2, using multilinear interpolation for the amplitude and frequency scaling coefficients, and *root macromodels*. Figs. 7-9 show the magnitude of the parametric macromodels of $\mathbf{S}_{11}(s, L, S)$ for the length values $L = \{10, 20\}$ mm and $\mathbf{S}_{14}(s, L, S)$ for the spacing value $S = 300$ μm. Figs. 10-13 compare $\mathbf{S}_{11}(s, L, S)$, $\mathbf{S}_{14}(s, L, S)$ and their macromodels for the length values $L = \{10.6, 19.4\}$ mm and the spacing value $S = 300$ μm that have not been used for the generation of the *root macromodels*. The worst case absolute error defined in (18) is equal to -18 dB and -40 dB for the old [4, 5] and new parametric macromodeling techniques, respectively.

Table IV. Number of poles of *root macromodels*.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	15	15	15	15	15	15
A_2	18	18	18	18	18	18
A_3	19	19	19	19	19	19
A_4	21	21	21	21	21	21
A_5	23	23	23	23	23	23
A_6	23	23	23	23	23	23
A_7	25	25	25	25	25	25

As in the previous example, the new proposed parametric macromodeling technique shows a better accuracy and modeling capability with respect to [4, 5]. It is able to accurately describe

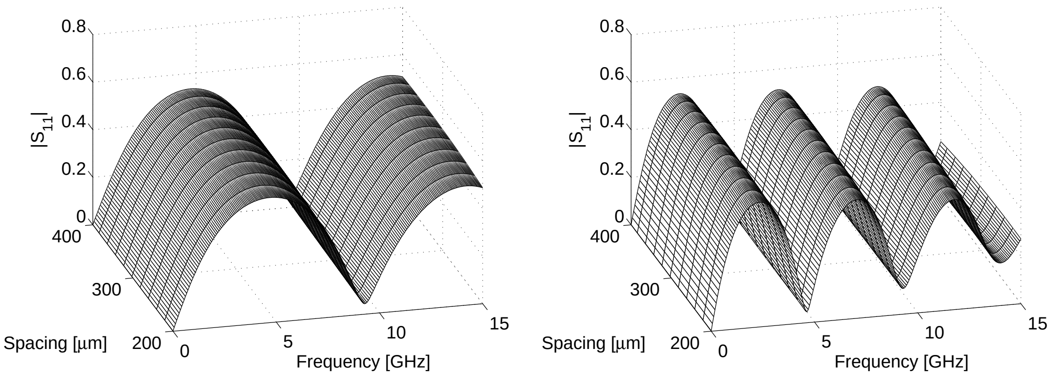


Figure 7. Magnitude of the trivariate macromodel of $\mathbf{S}_{11}(s, L, S)$ for $L = 10$ mm (left) and $L = 20$ mm (right) by means of the old technique [4, 5].

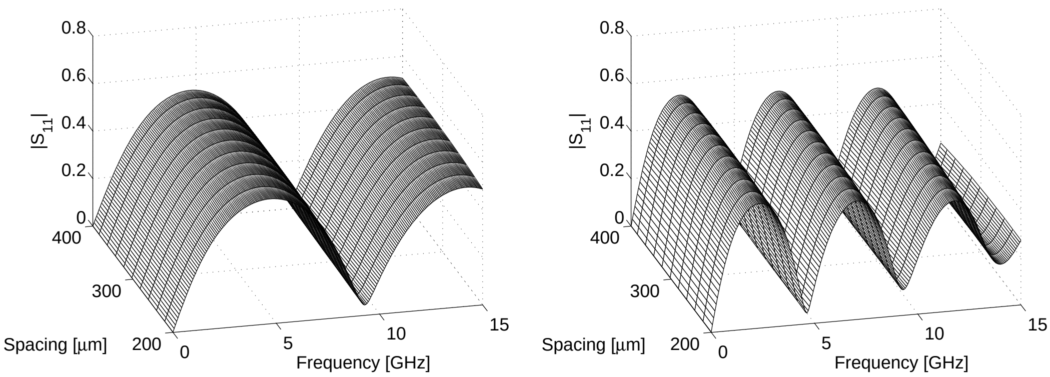


Figure 8. Magnitude of the trivariate macromodel of $\mathbf{S}_{11}(s, L, S)$ for $L = 10$ mm (left) and $L = 20$ mm (right) by means of the new technique .

the behavior of the system, while stability and passivity are guaranteed over the entire design space.

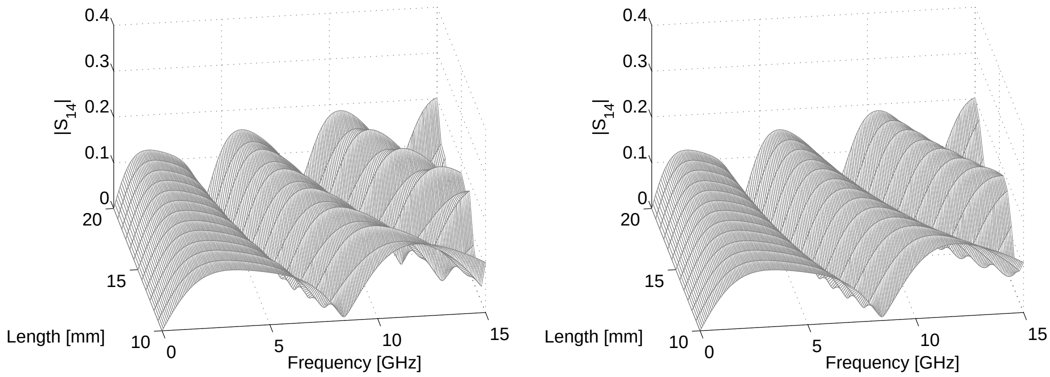


Figure 9. Magnitude of the trivariate macromodel of $\mathbf{S}_{14}(s, L, S)$ for $S = 300$ μm by means of the old technique [4, 5] (left) and the new technique (right).

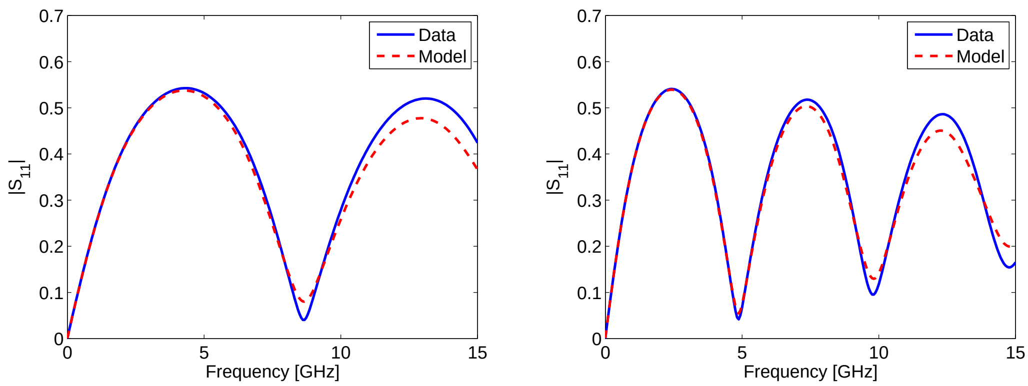


Figure 10. Magnitude of the trivariate macromodel of $\mathbf{S}_{11}(s, L, S)$ ($L = \{10.6, 19.4\}$ mm, $S = 300$ μm) by means of the old technique [4, 5].

4. Conclusions

We have presented a novel parametric macromodeling method for scattering representations. The overall stability and passivity of the parametric macromodel are guaranteed, while poles and residues are parameterized indirectly. The proposed method is based on an efficient and

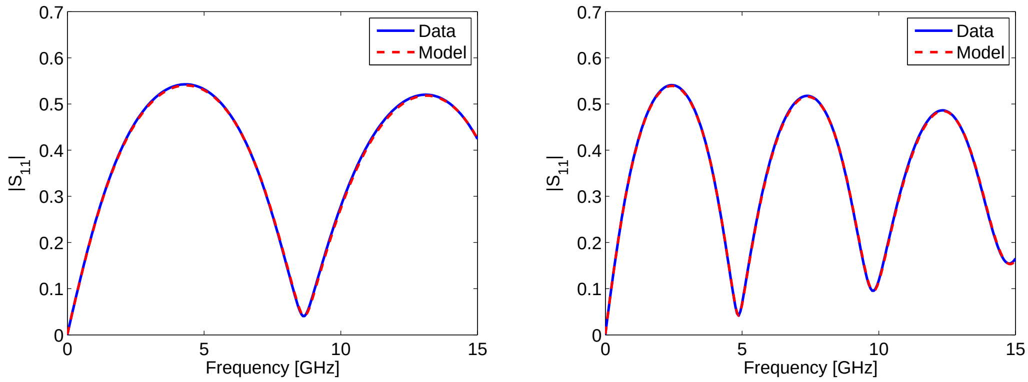


Figure 11. Magnitude of the trivariate macromodel of $\mathbf{S}_{11}(s, L, S)$ ($L = \{10.6, 19.4\}$ mm, $S = 300$ μm) by means of the new technique.

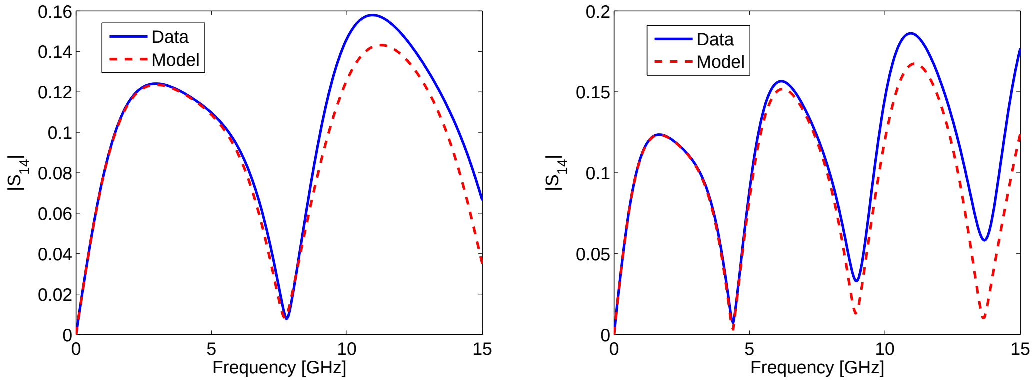


Figure 12. Magnitude of the trivariate macromodel of $\mathbf{S}_{14}(s, L, S)$ ($L = \{10.6, 19.4\}$ mm, $S = 300$ μm) by means of the old technique [4, 5].

reliable combination of rational identification, a procedure to find amplitude and frequency scaling system coefficients, and positive interpolation schemes. Numerical simulations confirm the capability of the proposed method of accurately describing dynamic electromagnetic systems, while preserving stability and passivity over the complete design space.

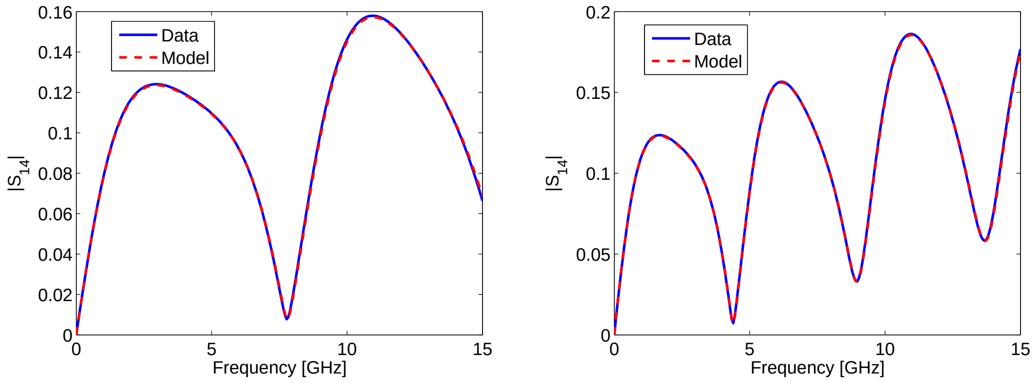


Figure 13. Magnitude of the trivariate macromodel of $\mathbf{S}_{14}(s, L, S)$ ($L = \{10.6, 19.4\}$ mm, $S = 300$ μm) by means of the new technique.

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